Problem 1 (Determinants): 6 POINTS  Show all Work

\[
A = \begin{bmatrix}
3 & 0 & 1 & 1 \\
1 & 0 & 2 & 1 \\
2 & 0 & 0 & 1 \\
-1 & 3 & 0 & 0
\end{bmatrix}
\]

(a) Calculate the determinant of \( A \) using row operations
(b) Calculate the determinant of \( A \) using cofactors expansion to check part (a)
(c) Do the columns of \( A \) span \( \mathbb{R}^4 \)? (Why or Why not)
Problem 2 (Inner Product): 8 POINTS  Show all Work

Let $f \cdot g = \frac{1}{2\pi} \int_{0}^{2\pi} f(\theta)g(\theta)d\theta$

a) Show $f \cdot g$ is an inner product in $\mathbb{C}^0$

b) Is $\sin(\theta)$ orthogonal to $\cos(\theta)$ with respect to the inner product above? If yes or no, Prove your answer is correct.

c) Find $\|\sin(\theta) - \cos(\theta)\|$

(This problem stresses the following point. Our results in class hold for any set of objects that form a vector space and for which we can define an inner product.)
Problem 2 (Work Space): Show all Work
Problem 3 (Eigenvalues): 6 POINTS  Show all Work

Given

\[ A = \begin{bmatrix} 3 & -2 & 8 \\ 0 & 5 & -2 \\ 0 & -4 & 3 \end{bmatrix} \]

Find the eigenvalues and corresponding eigenvectors for the matrix above.