1. (20 pts) Given the matrix

\[
A = \begin{pmatrix}
2 & -2 & 4 \\
1 & 0 & 3 \\
-1 & 3 & 0
\end{pmatrix}
\]

a) (10 pts) Compute an LU decomposition of A.

b) (10 pts) Solve Ax = b if

\[
b = \begin{pmatrix}
12 \\
9 \\
0
\end{pmatrix}
\]

2. (20 pts) Given the matrix

\[
A = \begin{pmatrix}
1 & -1 \\
-1 & 2 \\
-1 & 1 \\
0 & -1
\end{pmatrix}
\]

and the vector

\[
b = \begin{pmatrix}
1 \\
8 \\
-1 \\
1
\end{pmatrix}
\]

find the best solution to the equation Ax = b in the least squares sense.

3. (20 pts) Starting with the basis

\[
\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}
\]

use the Gram-Schmidt algorithm to produce an orthogonal basis.

4. (20 pts) Prove that if A is symmetric then all of its eigenvalues are real numbers.

5. (20 pts) ALWAYS/NEVER/SOMETIMES

a) Similar matrices have the same eigenvectors.

b) If B is symmetric and det(B)<0 then B has at least one real negative eigenvalue.

c) If A has full rank then A has only trivial null space.

d) An nxn orthogonal matrix has a trivial null space.

e) An upper triangular matrix has eigenvector e₁.

f) Det(-A) = -det(A)

g) A 3x3 matrix with eigenvalues {2, 1, -1} is invertible.

h) Row operations on a matrix preserve the determinant.

i) A 3x3 matrix with eigenvalues {2, 1, -1} is orthogonal.

j) If A is diagonal and has characteristic polynomial char(λ) then char(A)=0.