1. (20 pts) Given the matrix \( A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \)

   a) (10 pts) Compute the eigenvalues of \( A \).

   b) (10 pts) Compute the eigenvectors of \( A \).

2. (20 pts) Given the matrix \( A = \begin{pmatrix} 1 & 2 \\ 2 & -2 \\ -2 & 2 \end{pmatrix} \) and the vector \( b = \begin{pmatrix} -3 \\ 8 \\ 3 \end{pmatrix} \) find the best solution to the equation \( Ax = b \) in the least squares sense.

3. (20 pts) Starting with the set \( \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} \) use the Gram-Schmidt algorithm to produce an orthonormal set of vectors which spans the same subspace.

4. (20 pts) Prove that if \( Q \) is orthogonal and \( A \) is symmetric then \( Q^{-1}AQ \) is symmetric.

5. (20 pts) State whether each of the following is true in all cases, some cases or no cases: (ALWAYS/SOMETIMES/NEVER) Assume all matrices are real.

   a) If \( C = A + B \) then \( \det(C) = \det(A) + \det(B) \).

   b) \( \det(A^T A) \geq 0 \).

   c) A square matrix with a non-pivot column is onto.

   d) An invertible matrix is diagonalizable.

   e) If \( Q \) is orthogonal and \( Q^T AQ \) is diagonal then \( A \) is symmetric.

   f) \( \{x^2+1, x^2+x, x^2+x+1\} \) is a basis of \( \mathbb{P}_2 \). (TRUE/FALSE)

   g) If \( \begin{vmatrix} a & b & 0 \\ c & d & 1 \\ 0 & 5 & 0 \end{vmatrix} = 3 \) then \( \begin{vmatrix} a & b & 0 \\ c & d & 1 \\ 0 & 7 & 0 \end{vmatrix} = 5 \)

   h) The rank of a square matrix is the number of non-zero columns.

   i) Given: A is not diagonalizable. The eigenvalues of matrix A are distinct.

   j) Given: the matrix A is orthogonal. A is symmetric.