1. Let $X_1$ have the distribution function of a Bernoulli random variable (denote it by $F_1(x)$) and let $X_2$ have the distribution function of a Standard Normal random variable (denote it by $F_2(x)$). Show the for any two positive constants $a, b$ such that $a + b = 1$, $F(x) = aF_1(x) + bF_2(x)$ is a valid distribution function.

2. Let $X$ have the distribution function $F(x)$ given by

\[
F(x) = \begin{cases} 
0 & x < 0 \\
\frac{x+1}{4} & 0 \leq x < 1 \\
1 & 1 \leq x
\end{cases}
\]

Find the probability density of $Y = \log(x + 1)$.

3. For $X$ distributed as Binomial with parameters $n$ and $p$ show that $P[X \geq k] = P(Y \leq p)$ where $Y$ is a Beta random variable with parameters $k$ and $n - k + 1$.

4. Let $X_1$ and $X_2$ have the joint density function $f(x_1, x_2) = cx_1x_2^2$, $0 < x_1 < x_2 < 1$.

   (a) Find the value of the constant $c$.
   (b) Find the probability $P(X_1 + X_2 \leq 1/2)$.
   (c) Find the marginal densities of $X_1$ and $X_2$.
   (d) Find the marginal expectations and marginal variances of $X_1$ and $X_2$.
   (e) Find the correlation coefficient between $X_1$ and $X_2$. 