

STAT 451—Fall 2007

Practice Midterm II: Multivariate Distributions

Name: \_\_\_\_\_

1. Consider the joint probability mass function

	$X_1$	0	1	2
$X_2$				
1		0.2	0.1	0.1
2		0.2	0.0	0.1
3		0.0	0.0	0.3

- (a). Compute  $P(X_2 + X_1 > 3 | X_1 > 0)$ .
- (b). Compute  $E(X_1)$ ,  $E(X_2)$ ,  $Var(X_1)$  and  $Var(X_2)$ .
- (c). Compute the correlation coefficient between  $X_1$  and  $X_2$ .
- (d). Compute  $E[\max(X_1, X_2)]$ .

2. Let  $X$  be a random variable with mean  $\mu$  and moment generating function  $M_X(t)$ . Show that  $t^{-1} \log M_X(t) > \mu$ .

3. Suppose  $X$  and  $Y$  are independent *Exponential* random variables with parameter  $\lambda$ , i.e., they both have the density

$$f(x) = \frac{1}{\lambda} e^{-x/\lambda}, \quad x > 0, \quad \lambda > 0.$$

Let  $U = X + Y$  and  $V = X/Y$ .

- (a). Find the joint density of  $U$  and  $V$ .
- (b). Are  $U$  and  $V$  independent? Why?
- (c). Find the marginal density of  $U$  and  $V$ .

(d). Find  $E(V|U = 1)$ .

**4.** Let  $X$  and  $Y$  have the joint pdf  $f(x, y) = 2e^{-x-y}$ ,  $0 < x < y < \infty$ , zero elsewhere.

(a). Find the joint pdf of  $Z_1 = Y - X$  and  $Z_2 = 2X$ .

(b). Are  $Z_1$  and  $Z_2$  independent? Why?

(c). Find the pdf of  $Z_3 = Z_1 + Z_2$ .