

STAT 451—Fall 2007

Practice Final Exam: Some Special Distributions

Name: _____

1. [10] Suppose that X and Y are two bernoulli random variables. Prove that X and Y are independent if they are uncorrelated: $\text{Cov}(X, Y) = 0$.

2. [10] Let X_1 and X_2 be independent normal random variables with distribution $N(1, 1)$ and $N(2, 2)$, respectively. Find $P(3X_1 \leq 2X_2)$.

3. [10] Compute the skewness of the Poisson distribution with parameter λ . Skewness γ_1 is defined as

$$\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}},$$

where $\mu_r = E[(X - E(X))^r]$ is the r th central moment of X .

4. [10] A random variable X has a **Weibull distribution** if and only if its probability density is given by

$$f(x) = kx^{\beta-1}e^{-\alpha x^\beta}, \quad x > 0,$$

where $\alpha > 0$ and $\beta > 0$.

(a). Express k in terms of α and β .

(b). Show that $EX = \alpha^{-1/\beta}\Gamma(1 + \frac{1}{\beta})$.

5. [10] Let $X_1 \sim \text{Beta}(40, 20)$ and $X_2 \sim \text{Beta}(20, 20)$. Assume X_1 and X_2 are independent. Find the pdf of $Z = X_1X_2$.

6. [10] Let two independent random variables, X_1 and X_2 , be $N(0, 1)$ and $N(0, 1)$, respectively. Find the probability $P(|X_1| > |X_2|)$.