

STAT 451—Fall 2007

Practice Midterm I: Probability and Distributions

Name: _____ Please return this page with your solution after exam.

0. (5+5 Points) A box has 3 red balls numbered 1–3 and 2 blue balls numbered 1–2. Balls are drawn at random, without replacement until all balls are drawn.

- (a). What is the probability that the final ball drawn is a red ball?
- (b). What is the probability that neither the first ball nor the final ball drawn are red?

1. (10 Points) It is known that about 30% of human twins are identical and the rest are fraternal. Identical twins are necessarily the same sex — half are females and the other half are males. Furthermore, one-quarter of fraternal twins are both male, one-quarter are both female, and one-half are mixed (one male and one female).

- (a). What is the probability that the randomly selected twins are twin boys?
- (b). Given that a couple has just become the parents of twin girls, what is the probability the twins are identical?

2. (5+5+5 Points) A continuous random variable X that has the probability density function

$$f(x) = \frac{1}{2}e^{-|x|}, \quad -\infty < x < \infty.$$

- (a). Find the median of X , i.e., find x such that $P(X < x) = \frac{1}{2}$ and $P(X \geq x) = \frac{1}{2}$.
- (b). Find the moment generating function of X and use it to determine the mean and standard deviation of X .
- (c). Let $Y = |X|$. Find the probability density of Y .

3. (10 Points) The three major options on a certain type of new cars are an automatic transmission (A), a sunroof (B), and a stereo with compact disc player (C). If 70% of all purchasers request A, 80% request B, 75% request C, 85% request A or B, 90% request

A or C, 95% request B or C, and 98% request A or B or C. Compute the probability of following events.

- a) The next purchaser will select none of the three options.
- b) The next purchaser will request only an automatic transmission and not either of the other two options.
- c) The next purchaser will select exactly one of these three options.
- d) Are events A and B independent?

4. (10 Points) If an event A is independent of itself, then show that for any other event B , $P(A|B)$ is either 0 or 1.

5. (10 Points) Suppose X is a continuous random variable with support $[1,5]$ and pdf

$$f(x) = \begin{cases} 0.5 - c|x - 3| & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Find c . Find a such that $P(X \leq a) = 0.25$.

Bonus. (5 Points) A real number is picked at random from $[0, 1]$. Verify that event A that the first decimal digit of the number picked is 5 is independent of event B that the third decimal digit is 1.