An arbitrary precision SDP solver

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1 SDPHAM6

SDPHAM6 is an arbitrary precision implementation of the homogeneous interior-point method for semidefinite programming from [1]. It uses AHO search direction and a Mehrotra type primal-dual interior-point strategy. A rank-2 update is employed so that the computation of homogeneous directions slightly more expensive than in the non-homogeneous case. However, the algorithm takes fewer iterations and usually outperforms its non-homogeneous counterparts in terms of total CPU time.

The semi-definite programming (SDP) problem in primal standard form is:

$$\begin{array}{lll} (P) & \text{minimize}_X & C \bullet X \\ & \text{subject to} & A_i \bullet X = b_i, \quad i = 1, 2, \ldots, m, \quad X \succeq 0, \end{array} \tag{1}$$

where all $A_i, C \in \mathcal{S}_n$, $b \in \mathbb{R}^m$ are given, $X \in \mathcal{S}_n$ is the primal variable, and $C \bullet X = \text{trace}(CX)$. This optimization problem is a convex optimization problem, since its objective function and constraints are convex. The dual problem of (1) is

$$\begin{array}{lll} (D) & \text{maximize}_y & b^T y \\ & \text{subject to} & \sum_{i=1}^{m} y_i A_i \preceq C, \end{array} \tag{2}$$

where $y \in \mathbb{R}^m$ is the vector of dual variables.

Initially developed for the high precision computation of positive definite Hankel matrices of optimal spectral condition, SDPHAM6 can be used for any semidefinite programming problem which can not be accurately solved using double precision arithmetic. SDPHAM6 is implemented in the arbitrary precision environment of Mathematica 6.0 and is capable of performing the numerical calculations in as many digits as needed (as oposed to the existing double precision SDP solvers that uses only 16 digits). Using this code we have been able to accurately compute n-by-n positive definite Hankel matrices of minimum spectral condition number up to $n = 100$. Previous attempts of computing such matrices reported have not be able to go beyond $n = 25$.

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1.1 Obtaining and installing SDPHAM6

In order to use SDPHAM6 you must have Mathematica 6.0 installed. A copy of OOLCP can be obtained from http://www.math.umbc.edu/∼potra/sdpham6/sdpham6.tar.gz.

Once a copy of SDPHAM6's source code has been obtain, extract the files using

```
> tar xzvf sdpham6.tar.gz
```

The file *sdpha.m* is a Mathematica package and has to be installed appropriately. For doing this, open Mathematica 6.0 and choose *File* and then *Install*. Install it as a package and use *Sdpha* for the *Install Name* edit box. Press *Finish*. To test the installation, open a new notebook within Mathematica and load the package using

```
<<Sdpha`
```

No error message indicates that the installation was successful.

Besides the Mathematica package file *sdpha.m*, the archive also contains the file *HankelReduced.m* that builds the semidefinite programming problems needed to find positive definite Hankel matrices of minimal condition number and calls SDPHAM6 solver to solve such problems. This file contains an example of how the solver is called, see the next section for more details.

1.2 Using SDPHAM6 for SDP

SDPHAM6 can be called to solve SDP problems in dual form (2). By using a primal-dual interior-point methods, SDPHAM6 automatically forms and solves the primal problem (1), and therefore there is no need to supply it to SDPHAM6 solver.

The syntax to the function *SDPHASolve* that solves SDP problems of form (2) is

```
SDPHASolve[AA, b, C, m]
```

The parameter AA is an array of matrices containing matrices A_i, $i = 1, 2, \ldots, m$ from SDP problem (2). The remaining three parameters have to contain the rest of the data of problem (2), b, C, and m, respectively. The matrices part of the array AA and the matrix C must have the same size. Also the array b must be of size m. Both sparse and dense formats for matrices are supported by the SDPHAM6 solver.

The function *FindHankel* from the file *HankelReduced.m* contains an example of how SDPHAM6 is called and how the various parameters of the function *SDPHASolve* are formed.

References

[1] N. Brixius, F. A. Potra, and R. Sheng. SDPHA: a MATLAB implementation of homogeneous interior-point algorithms for semidefinite programming. *Optimization Methods and Software*, 11&12:583–596, 1999.