A Generalized p-value Approach to Inference on Common Mean

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Abstract

In this paper we suggest a generalized p-value approach to derive tests and confidence intervals for the common mean of several univariate normal populations. The case of a multivariate normal distribution with a common component mean is also addressed.

Keywords and phrases: Cholesky decomposition, common mean, confidence interval, generalized p-value, maximum likelihood, Wishart distribution.

1 Introduction

In this paper we consider the problem of providing exact test and confidence interval of the common mean of several univariate normal populations on the basis of a generalized p-value approach. This problem has been extensively discussed in the literature [Graybill and Deal (1959), Khatri and Shah (1974), Norwood and Hinkelmann (1977), Bhattacharya (1980), Pal and Sinha (1996), Mitra and Sinha (2005)]. Some important aspects of estimation of the common mean from a decision theoretic point of view have also been addressed by several authors [Sinha and Mouqadem (1982), Sinha (1979, 1985), Zacks (1966, 1970), Pal and Lim (1997)]. Regarding exact confidence interval for the common mean, Jordan and Krishnamoorthy (1996) discussed procedures based on linear combinations of t and F statistics, while Yu, Sun and Sinha (1999) considered this problem based on combination of associated p-values. Some recent contributions from a generalized p-value approach are due to Krishnamoorthy and Lu (2003) and Lin and Lee (2005).

Suppose we have k independent normal populations with a common mean μ and variances σ_i^2 for $i = 1, 2, \ldots$,k. Let ($\bar{x}_i$,s_i^2) be the sample mean and the sample variance of ith population based on a sample of size n_i. Let $D_i = \bar{x}_i - \bar{x}_1$ for $i = 2, \ldots, k$. Let $\mathcal{C}_u$ be the general class of unbiased estimates of μ, defined as $\mathcal{C}_u = \{\hat{\mu}_\phi : \hat{\mu} = \bar{x}_1 + \sum_{i=2}^{k} D_i \phi_i(s_1^2, s_2^2, \ldots, s_k^2; D_2^2, \ldots, D_k^2)\}$. Also let $\mathcal{C}_0$ be a subset of the general class of unbiased estimates of μ, defined as $\mathcal{C}_0 = \{\hat{\mu}_\phi : \hat{\mu} = \sum_{i=1}^{k} \bar{x}_i \phi_i(s_1^2, s_2^2, \ldots, s_k^2)\}$, with $\sum_{i=1}^{k} \phi_i = 1$. Clearly $\mathcal{C}_0 \subset \mathcal{C}_u$. The standard Graybill-Deal estimate of μ is given by

$$\hat{\mu}_{GD} = \frac{\sum_{i=1}^{k} \frac{\bar{x}_i}{s_i^2}}{\sum_{i=1}^{k} \frac{1}{s_i^2}}$$

which obviously belongs to $\mathcal{C}_0$. Sinha and Mouqadem (1982) also considered several other unbiased estimates of μ. Four popular estimates of μ in the context of two independent normal distributions are given below.

$$\begin{aligned}
\hat{\mu}_1 &= \frac{\frac{\bar{x}}{s_1^2} + \frac{\bar{y}}{s_2^2}}{\frac{1}{s_1^2} + \frac{1}{s_2^2}} \; [Graybill - Deal, 1959] \\
\hat{\mu}_2 &= \bar{x} + D\frac{s_1^2 + D^2}{s_1^2 + s_2^2 + D^2} \; [Sinha - Mouqadem, 1982] \\
\hat{\mu}_3 &= \bar{x} + Dmin(0.5, \frac{s_1^2}{s_1^2 + s_2^2}) \; [Sinha, 1979] \\
\hat{\mu}_4 &= \bar{x} + D\frac{s_1}{s_1 + s_2} \quad [Sinha - Mouqadem, 1982]\,,.
\end{aligned}$$

It should be noted that all the above estimates belong to $\mathcal{C}_u$.

Our main object in this paper is to use the generalized p-value approach based on a very general form of the unbiased estimate of the common mean given above in both $\mathcal{C}_0$ and $\mathcal{C}_u$

to construct exact tests and confidence intervals for the common mean. This includes some results of Lin and Lee (2005) as a special case.

Our second objective in this paper is to consider a multivariate normal population with a common component mean and an unknown dispersion matrix, and provide exact tests and confidence intervals for the common mean based on the generalized p-value approach. This is based on the well known Cholesky decomposition of a Wishart matrix.

The organization of the paper is as follows. For the sake of completeness, we provide a brief account of the generalized p-value procedure in Section 2. Sections 3 and 4 contain main results under the two classes $\mathcal{C}_0$ and $\mathcal{C}_u$, respectively. In Section 5 we consider the case of the multivariate normal population. Some simulation results comparing several competing procedures for two univariate normal populations are provided in Section 6. This section also contains the simulation results providing the performance of a proposed procedure for a bivariate normal population with a common mean.

2 Generalized p-value and Generalized Confidence Intervals

The concept of Generalized p-value was first introduced by Tsui and Weerahandi (1989) to deal with statistical testing problems in which nuisance parameters are present and also it is difficult to find a nontrivial test with some optimal properties.

Let X be the random variable with density function, $f(X|\zeta)$ where $\zeta = (\theta, \eta)$ where θ is our parameter of interest and η is nuisance parameter(s). η could be a vector of parameters.

Suppose we have the following hypothesis to test:

$$H_0 : \theta \leq \theta_0 \quad vs. \quad H_1 : \theta > \theta_0,$$

where θ_0 is specified value.

Let x be the observed value of the random variable X. $T = T(X; x, \zeta)$ is said to be a generalized test statistic if following three properties hold.

1. For fixed x and $\zeta = (\theta_0, \eta)$, the distribution of $T(X; x, \zeta)$ is independent of nuisance parameter η.

2. $t_{obs} = T(x; x, \zeta)$ does not depend on unknown parameters.

3. For fixed x and η, $P[T(X; x, \zeta) \geq t]$ is either stochastically increasing or decreasing in θ for any given t.

Then generalized p-value is defined as

$$p = \sup_{\theta \leq \theta_0} P[T(X; x, \theta, \eta) \geq t] = P[T(X; x, \theta_0, \eta) \geq t]$$

where $t = T(x; x, \theta_0, \eta)$.

Under the same set up, let $T_c(X; x, \theta, \eta)$ satisfy the following conditions:

1. The distribution of $T_c(X; x, \theta, \eta)$ does not depend on any unknown parameters.

2. The observed value of $T_c(X; x, \theta, \eta)$ is free of the nuisance parameter.

Then, we say $T_c(X; x, \theta, \eta)$ is a generalized pivotal quantity, and if t_1 and t_2 are such that

$$P[t_1 \leq T_c(X; x, \theta, \eta) \leq t_2] = 1 - \alpha$$

then $\Theta = \{\theta : t_1 \leq T_c(X; x, \theta, \eta) \leq t_2\}$ is a generalized confidence interval with confidence coefficient $1 - \alpha$. For example, if $T_c(x; x, \theta, \eta) = \theta$, then $[T_c(x; \alpha/2), T_c(x; 1 - \alpha/2)]$ is $1 - \alpha$ confidence interval for θ, where $T_c(x; \alpha/2)$ stands for α^{th} quantile.

3 Common Mean Problem in $\mathcal{C}_0$

In this section we provide exact tests and confidence intervals for the common mean of k univariate normals based on unbiased estimates in $\mathcal{C}_0 = \{\hat{\mu}_\phi : \hat{\mu} = \sum_{i=1}^{k} \bar{x}_i \phi_i(s_1^2, s_2^2, \ldots, s_k^2)\}$, with $\sum_{i=1}^{k} \phi_i = 1$. Note that $\bar{X}_i = \sum_{j=1}^{n_i} X_{ij}/n_i \sim N(\mu, \sigma_i^2/n_i)$ and $\frac{V_i}{\sigma_i^2} \sim \chi^2_{n_i - 1}, i = 1, 2, \ldots, k$ where $V_i = \sum_{j=1}^{n} (X_{ij} - \bar{X}_i)^2 = (n_i - 1)S_i^2$. Clearly $V_i's$ and $\bar{X}_i's$ are independent. Let $\mathbf{V} = (V_1, V_2, \ldots V_k)$, and $\mathbf{v} = (v_1, \cdots, v_k)$ be the observed value of $\mathbf{V}$. Let $\bar{\mathbf{X}} = (\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_k)$, and $\bar{\mathbf{x}} = (\bar{x}_1, \cdots, \bar{x}_k)$ be the observed value of $\bar{\mathbf{X}}$.

For a given $\phi(.)$, let $\hat{\mu}^* = \sum_{i=1}^{k} \bar{X}_i \phi(\sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2) \sim N\left(\mu, \sum_{i=1}^{k} \frac{\sigma_i^2}{n_i} \phi_i^2(\sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2)\right)$.

Let $I = I\left(\sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2\right) = \sum_{i=1}^{k} \frac{\sigma_i^2}{n_i} \phi_i^2(\sigma_1^2, \sigma_2^2, \ldots \sigma_k^2)$. Then $\frac{(\hat{\mu}^* - \mu)}{\sqrt{I}} \sim N(0,1) = Z(\text{say})$. Finally, we define I^* as

$$I^*\left(\mathbf{V}, \mathbf{v}; \sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2\right) = \sum_{i=1}^{k} \frac{\sigma_i^2}{n_i} \frac{v_i}{V_i} \phi_i^2(\sigma_1^2 \frac{v_1}{V_1}, \sigma_2^2 \frac{v_2}{V_2}, \ldots, \sigma_k^2 \frac{v_k}{V_k})$$

. We are now ready to define a generalized pivotal quantity for confidence interval of common mean μ through $\hat{\mu}^*$ and I^* as

$$\begin{aligned} T_c(\bar{\mathbf{X}}, \mathbf{V}; \bar{\mathbf{x}}, \mathbf{v}, \mu, \sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2) &= \sum_{i=1}^{k} \bar{x}_i \phi_i(\sigma_1^2 \frac{v_1}{V_1}, \sigma_2^2 \frac{v_2}{V_2}, \ldots, \sigma_k^2 \frac{v_k}{V_k}) - \frac{\sqrt{I^*}(\hat{\mu}^* - \mu)}{\sqrt{I}} \\ &= \sum_{i=1}^{k} \bar{x}_i \phi_i(\sigma_1^2 \frac{v_1}{V_1}, \sigma_2^2 \frac{v_2}{V_2}, \ldots, \sigma_k^2 \frac{v_k}{V_k}) \\ &- Z \sqrt{\sum_{i=1}^{k} \sigma_i^2 \frac{v_i}{V_i} \phi_i^2(\sigma_1^2 \frac{v_1}{V_1}, \sigma_2^2 \frac{v_2}{V_2}, \ldots, \sigma_k^2 \frac{v_k}{V_k})}. \end{aligned}$$

It can be easily shown that the two properties of generalized pivotal quantity as mentioned in Section 2 are satisfied because the distribution of $T_c()$ does not depend on any unknown parameters and the observed value of $T_c(\bar{\mathbf{x}}, \mathbf{v}; \bar{\mathbf{x}}, \mathbf{v}) = \mu$. So for given $\bar{\mathbf{x}}$ and $\mathbf{v}$, T_c is a generalized pivotal quantity.

Consider the following hypothesis:

$$H_0 : \mu \leq \mu_0 \qquad vs. \qquad H_1 : \mu > \mu_0.$$

From T_c, we can derive a generalized test variable, T, as $T = T_c - \mu$. So first two properties are automatically satisfied. Since the distributions of the random variables $V_1, \ldots, V_k, Z$ are free of parameters, it can be shown that $P\left[T \leq t; \mu | \, \bar{\mathbf{x}}, \mathbf{v}\right] = P\left[T \leq \mu | \, \bar{\mathbf{x}}, \mathbf{v}\right]$ which is an increasing function of μ. Hence generalized p-value can be obtained as

$$p = P[T \leq t | \, \mu = \mu_0; \, \bar{\mathbf{x}}, \mathbf{v}] = P\left[T \leq \mu_0 | \, \bar{\mathbf{x}}, \mathbf{v}\right].$$

H_0 can be rejected if $p < \alpha$.

For two sided alternative, generalized p value can be defined as

$$p = 2 * min\{P\left[T < \mu_0 | \, \bar{\mathbf{x}}, \mathbf{v}\right], P\left[T > \mu_0 | \, \bar{\mathbf{x}}, \mathbf{v}\right]\}.$$

Following the discussion in Section 2, it is easy to construct a two-sided confidence interval for μ , which is given by $[T_c(x;\alpha/2),\ T_c(x;1-\alpha/2)]$ with confidence level $1-\alpha$. Here $T_c(x;\alpha/2)$ stands for α^{th} quantile of the relevant pivot T_c.

4 Common Mean Problem in $\mathcal{C}_u$

Let $\hat{\mu}^* = \sum_{i=1}^{k} \bar{X}_i \phi_i \left(\sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2; \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}, \ldots, \frac{\sigma_1^2}{n_1} + \frac{\sigma_k^2}{n_k}\right)$. Obviously,
$\hat{\mu}^* \sim N\left(\mu, \sum_{i=1}^{k} \frac{\sigma_i^2}{n_i} \phi_i^2 \left(\sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}, \ldots, \frac{\sigma_1^2}{n_1} + \frac{\sigma_k^2}{n_k}\right)\right)$.
Let

$$\begin{aligned} I &= I\left(\sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}, \ldots, \frac{\sigma_1^2}{n_1} + \frac{\sigma_k^2}{n_k}\right) \\ &= \sum_{i=1}^{k} \frac{\sigma_i^2}{n_i} \phi_i^2 \left(\sigma_1^2, \sigma_2^2, \ldots \sigma_k^2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}, \ldots, \frac{\sigma_1^2}{n_1} + \frac{\sigma_k^2}{n_k}\right). \end{aligned}$$

Then clearly $\frac{(\hat{\mu}^* - \mu)}{\sqrt{I}} \sim N(0,1) = Z(\text{say})$. We now define I^* as

$$\begin{aligned} & I^*\left(\mathbf{V}, \mathbf{v}; \sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2\right) \\ &= \sum_{i=1}^{k} \frac{\sigma_i^2}{n_i} \frac{v_i}{V_i} \phi_i^2 \left(\sigma_1^2 \frac{v_1}{V_1}, \sigma_2^2 \frac{v_2}{V_2}, \ldots, \sigma_k^2 \frac{v_k}{V_k}, \frac{\sigma_1^2}{n_1} \frac{v_1}{V_1} + \frac{\sigma_2^2}{n_2} \frac{v_2}{V_2}, \ldots, \frac{\sigma_1^2}{n_1} \frac{v_1}{V_1} + \frac{\sigma_k^2}{n_k} \frac{v_k}{V_k}\right). \end{aligned}$$

Now using similar technique as in preceding section, we can define a generalized pivotal quantity for confidence interval of the common mean μ by

$$
\begin{aligned}
& T_c(\bar{\mathbf{X}}, \mathbf{V}; \bar{\mathbf{x}}, \mathbf{v}, \mu, \sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2) \\
&= \sum_{i=1}^{k} \bar{x}_i \phi_i \left(\sigma_1^2 \frac{v_1}{V_1}, \sigma_2^2 \frac{v_2}{V_2}, \ldots, \sigma_k^2 \frac{v_k}{V_k}, \frac{\sigma_1^2}{n_1}\frac{v_1}{V_1} + \frac{\sigma_2^2}{n_2}\frac{v_2}{V_2}, \ldots, \frac{\sigma_1^2}{n_1}\frac{v_1}{V_1} + \frac{\sigma_k^2}{n_k}\frac{v_k}{V_k} \right) - \frac{\sqrt{I*}(\hat{\mu}^* - \mu)}{\sqrt{I}} \\
&= \sum_{i=1}^{k} \bar{x}_i \phi_i \left(\sigma_1^2 \frac{v_1}{V_1}, \sigma_2^2 \frac{v_2}{V_2}, \ldots, \sigma_k^2 \frac{v_k}{V_k}, \frac{\sigma_1^2}{n_1}\frac{v_1}{V_1} + \frac{\sigma_2^2}{n_2}\frac{v_2}{V_2}, \ldots, \frac{\sigma_1^2}{n_1}\frac{v_1}{V_1} + \frac{\sigma_k^2}{n_k}\frac{v_k}{V_k} \right) \\
&- Z \sqrt{\sum_{i=1}^{k} \frac{\sigma_i^2}{n_i}\frac{v_i}{V_i} \phi_i^2 \left(\sigma_1^2 \frac{v_1}{V_1}, \sigma_2^2 \frac{v_2}{V_2}, \ldots, \sigma_k^2 \frac{v_k}{V_k}, \frac{\sigma_1^2}{n_1}\frac{v_1}{V_1} + \frac{\sigma_2^2}{n_2}\frac{v_2}{V_2}, \ldots, \frac{\sigma_1^2}{n_1}\frac{v_1}{V_1} + \frac{\sigma_k^2}{n_k}\frac{v_k}{V_k} \right)}.
\end{aligned}
$$

It should be noted that the distribution of T_c does not depend on any unknown

parameters and the observed value, i.e., $T_c(\bar{\mathbf{x}}, \mathbf{v}; \bar{\mathbf{x}}, \mathbf{v})$ is equal to μ. So T_c is a generalized pivotal quantity. Hence we can find p-value for hypothesis testing problem as described in the preceding section.

5 Multivariate normal set up

Suppose we have n observations, $(\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n)$, from a k variate normal distribution given by $N(\mu\mathbf{1}, \Sigma)$ where μ is the common mean. Let $S = \sum_{i=1}^{n}(\mathbf{X}_i - \bar{\mathbf{X}})(\mathbf{X}_i - \bar{\mathbf{X}})'$ be the matrix of sample sum of squares and products, which follows a Wishart distribution with parameters $m = n - 1$ and Σ.

If Σ is known, the maximum likelihood estimate (MLE) of μ is given by $\hat{\mu}^* = \frac{\mathbf{1}'\Sigma^{-1}\bar{\mathbf{X}}}{\mathbf{1}'\Sigma^{-1}\mathbf{1}}$. When Σ is unknown, we can replace it by $S/(n-1)$ which gives $\hat{\mu} = \frac{\mathbf{1}'S^{-1}\bar{\mathbf{x}}}{\mathbf{1}'S^{-1}\mathbf{1}}$. It is shown in Halperin (1961) that $\hat{\mu}$ is MLE.

Although $\hat{\mu}$ is unbiased for μ, it is obvious that the variance of $\hat{\mu}$ and its exact distribution are quite complicated, and hence it is not obvious how one should proceed to provide *exact* inference for μ based on $\hat{\mu}$. Exact tests and confidence intervals for μ can also be constructed using some properties of a multivariate normal distribution [see section 6]. Our goal here is to provide an exact test and confidence interval for μ, utilizing $\hat{\mu}$ based on generalized p value approach. Exact tests and confidence intervals can also be constructed using some properties of the underlying multivariate normal distributions. Details appear in Krishnamoorthy and Lu (2005).

By Cholesky Decomposition, we can find an upper triangular matrix C such that $\Sigma = C'C$. Hence $\Sigma^{-1} = C^{-1}C'^{-1}$. It easy to show that when Σ is known, $\hat{\mu}^* \sim N\left(\mu, \frac{1}{n\mathbf{1}'\Sigma^{-1}\mathbf{1}}\right)$. Hence

$$Z = \sqrt{n\mathbf{1}'\Sigma^{-1}\mathbf{1}}(\hat{\mu}^* - \mu) \sim N(0,1).$$

As S is a symmetric positive definite matrix, we can find an upper triangular matrix A such that $S = A'A$. Let s, a be the observed matrix of S and A, respectively. We can represent S as $S = \sum_{\alpha=1}^{n-1} \mathbf{z}_\alpha \mathbf{z}'_\alpha$ where $\mathbf{z}_\alpha \sim N(0,\Sigma)$. So $C'^{-1}SC^{-1} = \sum_{\alpha=1}^{n-1}(C'^{-1}\mathbf{z}_\alpha\mathbf{z}'_\alpha C^{-1})$ where $C'^{-1}\mathbf{z}_\alpha \sim N(0, I_k)$ as $\Sigma = C'C$. Then

$$C'^{-1}A'AC^{-1} = B'B \sim Wishart(n-1, I_k)$$

where $B = AC^{-1}$. Now from *Theorem* 3.2.14 in Muirhead (1982), since B is an upper triangular matrix, we can conclude that

$$B_{ii}^2 \sim \chi^2_{n-i} \quad \text{and} \quad B_{ij} \sim N(0,1).$$

Here $B = ((B_{ij}))$ and all B_{ij}'s are independent. Following the same technique as before, we can define a generalized pivotal quantity, T_c, as

$$\begin{aligned} T_c &= \frac{\mathbf{1}'a^{-1}AC^{-1}C'^{-1}A'a'^{-1}\bar{\mathbf{x}}}{\mathbf{1}'a^{-1}AC^{-1}C'^{-1}A'a'^{-1}\mathbf{1}} - \frac{\sqrt{n\mathbf{1}'\Sigma^{-1}\mathbf{1}}(\hat{\mu}^* - \mu)}{\sqrt{n\mathbf{1}'a^{-1}AC^{-1}C'^{-1}A'a'^{-1}\mathbf{1}}} \\ &= \frac{\mathbf{1}'a^{-1}BB'a'^{-1}\bar{\mathbf{x}}}{\mathbf{1}'a^{-1}BB'a'^{-1}\mathbf{1}} - \frac{Z}{\sqrt{n\mathbf{1}'a^{-1}BB'a'^{-1}\mathbf{1}}}. \end{aligned}$$

It is easy to show that T_c satisfies the conditions of being a pivotal quantity. The distribution of T_c does not depend on any parameter and its observed value is μ. Hence we can use this to construct a generalized confidence interval for μ.

If we write $T = T_c - \mu$, then we can claim by following previous techniques that T is indeed a generalized test variable which can be used to compute generalized p-value.

Consider the following hypothesis:

$$H_0 : \mu \leq \mu_0 \qquad vs. \qquad H_1 : \mu > \mu_0.$$

From T_c, we can derive a generalized test variable T as $T = T_c - \mu$. So first two properties are automatically satisfied. Since distributions of the random variables $V_1, \ldots, V_k, Z$ are free of parameters, it can be shown that $P\left[T \leq t; \mu | \, \bar{\mathbf{x}}, s\right] = P\left[T \leq \mu | \, \bar{\mathbf{x}}, s\right]$ which is an increasing function of μ. Hence generalized p-value can be obtained as

$$p = P[T \leq t | \, \mu = \mu_0; \, \bar{\mathbf{x}}, s] = P\left[T \leq \mu_0 | \, \bar{\mathbf{x}}, s\right].$$

H_0 can be rejected if $p < \alpha$.

6 Simulated coverage probabilities and powers

In this section we report the simulated coverage probabilities and powers for each method based on estimates in section 1 for two univariate normal populations. We also report simulated coverage probabilities and powers of the proposed method suggested in section 5 under a bivariate normal set up. In each case, 10000 iterations were performed to compute coverage probability and powers. We take $\sigma_1^2 = 1$ in all computation. We generate samples under $H_0 : \mu = 0$.

Under two independent univariate normals $\mu = 1$ is taken in order to simulate powers for the methods 1,2,3,4 (shown in table 1) corresponding to the four estimates mentioned in section 1, and $\mu = 0.5$ is taken to simulate power for bivariate normal set up (shown in table 2). All figures are computed by considering two sided confidence intervals and powers for two sided alternative.

Table 1: Simulated coverage prob. and powers for four methods

Method		1		2		3		4	
σ_2^2	n	Cov.prob	Power	Cov.prob	Power	Cov.prob	Power	Cov.prob	Power
1	5	0.93	0.77	0.93	0.75	0.98	0.09	0.93	0.06
	10	0.94	0.97	0.93	0.90	0.98	0.12	0.94	0.05
	15	0.94	0.99	0.94	0.97	0.98	0.15	0.94	0.04
1.5	5	0.93	0.72	0.93	0.75	0.97	0.06	0.94	0.07
	10	0.94	0.95	0.93	0.91	0.98	0.10	0.94	0.04
	15	0.94	0.99	0.94	0.97	0.98	0.16	0.94	0.05
2	5	0.93	0.68	0.93	0.76	0.98	0.06	0.93	0.07
	10	0.94	0.93	0.93	0.92	0.98	0.10	0.94	0.06
	15	0.94	0.98	0.94	0.97	0.99	0.17	0.94	0.05

It is evident that method 1 and method 2 are performing very well compared to the other two methods from the point of view of power. But all these methods are reasonable in terms of coverage probability. Sometimes method 2 is better than method 1 in terms of power. But in terms of coverage probability method 3 is better than all other methods to some extent.

Table 2: Simulated coverage prob. and power for bivariate normal

		$n = 5$		$n = 10$		$n = 15$	
σ_2^2	ρ	Cov.prob	Power	Cov.prob	Power	Cov.prob	Power
1	0	0.82	0.19	0.67	0.54	0.55	0.81
	0.25	0.80	0.24	0.62	0.58	0.52	0.80
	0.50	0.74	0.28	0.59	0.59	0.50	0.80
	0.75	0.72	0.30	0.58	0.58	0.49	0.77
1.25	0	0.80	0.21	0.66	0.54	0.56	0.79
	0.25	0.78	0.25	0.60	0.57	0.52	0.80
	0.50	0.74	0.29	0.59	0.58	0.50	0.78
	0.75	0.72	0.32	0.57	0.57	0.50	0.77
1.5	0	0.80	0.21	0.65	0.52	0.55	0.78
	0.25	0.77	0.25	0.60	0.58	0.51	0.78
	0.50	0.74	0.28	0.58	0.57	0.49	0.76
	0.75	0.72	0.31	0.58	0.58	0.49	0.75
1.75	0	0.79	0.21	0.65	0.53	0.56	0.76
	0.25	0.77	0.26	0.60	0.57	0.50	0.77
	0.50	0.74	0.29	0.58	0.58	0.49	0.77
	0.75	0.72	0.30	0.57	0.58	0.49	0.75
2	0	0.79	0.22	0.64	0.53	0.54	0.75
	0.25	0.76	0.26	0.60	0.56	0.52	0.77
	0.50	0.73	0.30	0.58	0.57	0.49	0.75
	0.75	0.72	0.31	0.57	0.57	0.47	0.75

This method is performing well in terms of powers but it is not so satisfactory from the point of view of coverage probability. Also as ρ increases, power decreases.

This method can be used in multivariate (dimension ≥ 3) normal distribution. It might perform in a better way for the normal distribution with dimension more than 2.

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