1. (20) Prove by induction that \( \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4} \).
2. (20) Let $A$ be a nonempty set of real numbers which is bounded below. Let $-A$ be the set of all numbers $-x$, where $x \in A$. True or False? $\inf A = - \sup(-A)$. 

Mark one and explain  o  False  o  True
3. (20) True or False? Let $f : \mathbb{R} \to \mathbb{R}$, and suppose that $|f(x) - f(y)| \leq (x - y)^2$ for each pair of real numbers $x, y$. Does this imply that $f$ is constant?

Mark one and explain ◯ False ◯ True
4. (20) True or False? Let $f : [a, b] \to \mathbb{R}$ be a (not necessarily continuous) function with the property that

$$\forall x \in [a, b] \text{ there is } \delta_x > 0 \text{ and } m_x > 0 \text{ such that } |f(s)| < m_x \text{ if } |s - x| < \delta_x.$$ 

Does this imply that $f$ is bounded on $[a, b]$?

Mark one and explain ✗ False    ✗ True
5. (20) True or False? A function $f : \mathbb{R} \to \mathbb{R}$ is said to be convex if $f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y)$. Does this imply that $f$ is continuous?

Mark one and explain ☐ False ☐ True
6. (20) True or False? Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$. If $f(r) = 0$ for each $r$ rational, then $f(x) = 0$ for each $x \in \mathbb{R}$.

Mark one and explain $\square$ False $\quad \square$ True
7. (20) True or False? If $f : (a, b) \to \mathbb{R}$ is uniformly continuous, then $f$ is bounded on $(a, b)$.
8. (20) True or False? Let \( f(x) = x \), and \( g(x) = \sin x \). (Hint: \(|\sin x| \leq |x|\), and \( \sin a - \sin b = -2 \sin \left( \frac{a-b}{2} \right) \cos \left( \frac{a+b}{2} \right) \).) Mark one and explain

(a) (2) \( f(x) \) is uniformly continuous on \( \mathbb{R} \). □ False □ True

(b) (8) \( g(x) \) is uniformly continuous on \( \mathbb{R} \). □ False □ True

(c) (10) \( f(x)g(x) \) is uniformly continuous on \( \mathbb{R} \). □ False □ True
9. (20) Let \( f : \mathbb{R} \to \mathbb{R} \) be given by \( f(x) = x|x| \). Find \( f'(x) \) when \( f \) is differentiable and describe the set \( D \subseteq \mathbb{R} \) where \( f'(x) \) exists.

\[
D \text{ is } \quad f'(x) =
\]
10. (20) True or False? Let \( f : (0, \infty) \to \mathbb{R} \) be differentiable. If \( f'(x) \to b \) as \( x \to \infty \), then

\[
\lim_{x \to \infty} \frac{f(x)}{x} = b.
\]

Mark one and explain: False, True