Advances in Control of Partial Differential Equations

A Conference in Honor of Thomas I. Seidman

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Department of Mathematics and Statistics

University of Maryland, Baltimore County (UMBC)

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Abstracts of the Invited Presentations

1. **Roger W. Brockett**, *Division of Engineering and Applied Sciences, Harvard University*

   The Influence of Continuum Dynamics on the Stabilization Problem

   The problem of stabilizing an unstable system by feedback is frequently encountered in applications of control. Although unsolved problems arise in a nonlinear setting, for controllable finite dimensional linear systems the stabilization problem is generally felt to be a well understood. Even so, in experimental settings difficulties can arise. In this talk I will build on various known results on singular control to show that a constraint on the power that can be delivered by the actuator frequently can result in a surprisingly tight restriction on the achievable domain of attraction and that continuum effects such as high frequency elastic modes, typically have a significant effect on the extent of such a domain. Some experimental results involving an inverted double pendulum will be discussed.

2. **Irena Lasiecka**, *Department of Mathematics, University of Virginia*

   Blowup Rates for the Minimal Energy Associated with Null-Controllability of Coupled PDE Systems

   Starting with a 1988 influential paper by Tom Seidman "How violent are fast controls?", the issue of optimal asymptotics for blowup rates associated with observability estimates in control systems, that can be controlled in an arbitrary short time, became of great and — in fact — of still increasing interest. Systems that can be controlled (observed) in an arbitrary short time, exhibit, by necessity, blowup of "the cost of control" when the control (observation) time $T \to 0$. It is thus of interest to know how "fast" ("violent") this blowup is. The 1988 paper by Tom Seidman provides sharp and optimal answer to this question — in the case of finite dimensional control systems. In fact, the blowup rates established there have one to one correspondence with Kalman's rank condition.

   It turned out, many years later, that blowup asymptotics are critical in the study of elliptic degenerate equations and also in the study of regularity of stochastic processes such as Orstein Uhlenbeck process. These discoveries have provided for renewed interest and activities in the field. Since the finite dimensional case has been completely solved in Tom's 1988 paper, the attention was directed toward control systems governed by partial differential equations. And here, again, an "old" Tom Seidman 1984 paper "Two results on exact boundary control of parabolic equations" came to the rescue. This paper gave, for the very first time, optimal asymptotics for one dimensional heat equation with boundary control.

   The goal of my talk is to present subsequent developments in this area, which also include some of the joint work with Tom. Particular emphasis will be paid to blowup asymptotic arising in *systems* of PDEs. In fact, coupled PDEs, with control acting only on one component of the system, present particular challenge, as they combine finite-dimensional difficulties compiled by the infinite-dimensional character of PDE structure involved.

3. **Suzanne Lenhart**, *Department of Mathematics, University of Tennessee and Oak Ridge National Laboratory*

   Optimal Control of Integrodifference Population Models

   Integrodifference equations are models that are discrete in time and continuous in space. These equations model populations with discrete generations with separate growth and dispersal stages. The dispersal is modeled by an integral of the population density (after the growth) against a kernel. Analysis and characterization of an optimal harvesting control example will be given using an adjoint equation. Numerical illustrations will be included for a variety of dispersal kernels and applications (including gypsy moth pest control and plant pathogen model).

4. **Boris S. Mordukhovich**, *Department of Mathematics, Wayne State University*

   Optimal Control of Evolution Systems

   This talk is devoted to optimal control problems governed by evolution/differential inclusions in finite-dimensional and infinite-dimensional spaces and also by semilinear partial differential inclusions. We pursue a twofold goal: to develop the method of discrete approximations for such problems and to derive necessary optimality conditions of the Euler-Lagrange type under natural assumptions. First we consider a generalized Bolza problems for infinite-dimensional differential inclusions with endpoint constraints. One of the principle differences between finite-dimensional and infinite-dimensional dynamic systems is the lack of compactness in infinite dimensions. Constructing well-posed discrete approximations and using advanced tools of variational analysis and generalized differentiation, we derive necessary conditions for discrete-time problems and then, by passing to the limit, for continuous-time evolution inclusions. A similar procedure is developed for optimal control problems of the Mayer type governed by constrained semilinear inclusions with unbounded operators generating compact semigroups. This particularly covers parabolic partial differential inclusions whose solutions are understood in the conventional mild sense.

5. **David Russell**, *Department of Mathematics, Virginia Tech*

   An Overview of Control of Elastic Systems

   This talk will summarize work undertaken over the last decade toward developing a static control theory for elastic systems. We will briefly develop the setting of the problem, indicate how the standard control theory concepts are modified for this setting, and indicate results thus far in hand. We will end our talk by describing some of the open questions in the subject, anticipating more or less instantaneous solutions from the audience.

6. **Héctor J. Sussmann**, *Department of Mathematics, Rutgers University*

   Set Separation Theorems and the Pontryagin Maximum Principle: A Survey

   It is well known that whenever a trajectory is a solution of an optimal control problem or a Pareto optimization problem, possibly with state space constraints, this implies that two sets (an "atttainable set" A and an "unwanted set" U) are locally separated near a certain point p (in the sense that, for some neighborhood V of p, the intersection $A \cap U \cap V$ is empty). Thus, necessary conditions for local set separation give rise to necessary conditions for optimality. One such condition says, roughly, that if two subsets S_1 and S_2 of R^n have "tangent cones" C_1 and C_2 at a point $p \in S_1 \cap S_2$, then strong transversality of the cones C_1, C_2 implies that the intersection $S_1 \cap S_2$ contains points other than p arbitrarily close to p, so in particular S_1 and $S_2 \backslash \{p\}$ are not locally separated at p. (Two convex cones C_1, C_2 are strongly transversal if (i) the set $\{c_1 - c_2 : c_1 \in C_1, c_2 \in C_2\}$ is the whole space, and (ii) the intersection $C_1 \cap C_2$ contains at least a full half-line.) Hence a necessary condition for local separation of A and U at p, if p does not belong to $A \cap U$, is that their "tangent cones" C_1 and C_2 not be strongly transversal. This in turn leads to the existence of the usual adjoint covector of the Maximum Principle. The problem with the above somewhat vague statement is that, to make it precise, one needs a precise notion of "tangent cone." There are many such notions, and not all of them have the property that strong transversality implies non-separation. One family of "tangent cone" definitions consists of generalizations of the old notion of a Boltyanskii approximating cone, while another family involves generalizations of the Clarke tangent cone. Versions of the Maximum Principle corresponding to each of the two families have been proved in recent years, but no proof exists of a version in which, for example, the attainable set has a "tangent cone" in the Boltyanskii sense and the unwanted set has a "tangent cone" in the Clarke sense. The explanation of this strange fact was discovered in January of 2006 by Alberto Bressan, who constructed a counterexample showing that, when C_1 is a Boltyanskii cone and C_2 is a Clarke cone, strong transversality of the cone can occur while the sets themselves are locally separated at p. This discovery has important consequences, which will be discussed in detail in the talk.

7. **Roberto Triggiani**, *Department of Mathematics, University of Virginia*

   Control problems for Navier-Stokes equations and fluid-structure interaction

   This talk will present a set of problems either by the speaker or jointly with other co-authors on the stabilization of Navier Stokes equations as well as on semigroup well-posedness, spectral analysis and stability of a fluid-structure interaction.

8. **Jiongmin Yong**, *Department of Mathematics, University of Central Florida*

   Second-Order Spike Variations for Controlled Partial Differential Equations

   For optimal control problems (both for ordinary differential equations, and partial differential equations), when the control domain is just a metric space (which implies that no convexity condition is assumed), to derive necessary conditions for optimal controls, one has to introduce the so-called spike variation technique. Recently, there are some authors working on second order necessary (and/or sufficient) conditions for optimal controls, with the crucial assumption that the control domain is a convex set. It is natural to ask what happens if the control domain is not necessarily convex. In this paper, we make some efforts to introduce a kind of second order spike variations for controlled partial differential equations. With such a technique, second-order necessary conditions are derived for optimal controls of partial differential equations with the control domain not necessarily convex.

Abstracts of the Poster Presentations

1. **Jason Adaska**, *Division of Engineering and Applied Sciences, Harvard University*

   Optimal Traffic Shaping in a Fluid Queue

   A non-work-conserving queuing policy is one in which the server does not always work at the maximum allowable rate. In a queuing network, such policies are often employed to "shape" the traffic. This generally means reducing the variation of the traffic leaving the queue so that it is more manageable for downstream nodes. Here we approach traffic shaping from a control theoretic point of view and compute the optimum traffic shaping policy. Our queuing model is a finite buffer size, fluid queue which balances two factors : reducing rate-jitter and avoiding buffer overflow. We model our queuing system with stochastic differential equations driven by Poisson counters, and derive the Hamilton-Jacobi-Bellman optimality equations. We numerically compute the optimum traffic-shaping control for an example system and compare it to its work conserving version.

2. **Lorena Bociu**, *Department of Mathematics, University of Virginia*

   Existence, Uniqueness and Blow-Up of Solutions to Wave Equations with Supercritical Boundary Sources and Boundary Damping

   In this poster, I would like to summarize the wellposedness and blow-up results that I have obtained for the wave equation driven by boundary sources with critical and supercritical exponents. Taking advantage of the boundary damping, local and global existence of finite energy solutions are obtained, while in the absence of boundary "over-damping", finite time blow-up of weak solutions is exhibited. The results extend those obtained recently in the literature, where boundary sources and dampings of polynomial structures with subcritical exponents were considered. In addition, the local and global existence theory presented is sharp, since the blow-up phenomenon is exhibited in the complementary (to global existence) region.

   This work is joint with Irena Lasiecka, University of Virginia.

3. **Paul Cokeley**, *Department of Mathematics, University of Nebraska, Lincoln*

   Boundary Controllability of Nonlinear Structurally Damped Elastic Systems

   This poster provides results of boundary control for 2-D structurally damped systems in the presence of specific nonlinearities. The plate satisfies hinged boundary conditions that will steer the dynamics to null state at the terminal time. This terminal time will be governed by size of given initial data. The proof technique is similar to that of [G. Avalos, Null Controllability of von Karman thermoelastic plates under the clamped or free mechanical boundary conditions, J. Math. Anal. Appl. 318 (2006) 410–432]. The work presented also relies heavily on estimates for the minimal energy function for the linearized structurally damped system provided in [G. Avalos and P. Cokeley, Boundary and Local Null Controllability of Structurally Damped Elastic Systems, to appear].

4. **Wandi Ding**, *Department of Mathematics, University of Tennessee, Knoxville*

   Optimal Harvesting of a Spatially Explicit Fishery Model

   We consider an optimal fishery harvesting problem using a semilinear elliptic PDE model, which has logistic growth and the harvest depends on the location of the fish. We consider two objective functionals: maximizing the yield and minimizing the cost or the variation in the fishing effort (control). Existence, necessary conditions and uniqueness for the optimal harvesting control for both cases are established. The optimal control when minimizing the variation is characterized by a variational inequality instead of the usual algebraic characterization. In both cases, we need to solve the optimality system of nonlinear elliptic partial differential equations. Numerical examples are given to illustrate the results.

   This work is joint with Suzanne Lenhart, University of Tennessee, Knoxville and Oak Ridge National Laboratory.

5. **Afshin Izadian**, *Lane Department of Computer Science and Electrical Engineering, West Virginia University*

   Nonlinear Control of MEMS-Gyroscopes

   MEMS-base vibratory rate gyroscopes ideally consist of un-coupled orthogonal drive and sense axes in a common plane. The drive axis is driven at its resonant frequency while the sense axis remains stationary until a rotation on the common plane generates Coriolis force which transfers energy from the drive axis to the sense axis. The amount of transferred energy is related to the rate of change of the rotation angle. Differential equations governing the motion of MEMS gyroscopes often do not account for interdependency between sense and drive axes dynamics. However, in real systems, due to imperfect manufacturing steps and environmental operating conditions, the drive and sense axes are somehow interconnected through their mutual dependencies. Anisoelasticity of the manufactured devices causes an interrelation between drive and sense axis spring constants, and adds additional mutual terms to the dynamics of the system. The operating conditions in the package environment result in similar damping coefficient interrelation, known as anisoinertia. The presence of these anisoelasticity and anisoinertia terms causes an unwanted noise signal, or drift, on the sense axis. This noise can be either in phase or out of phase with the sensed signal, and is present for stationary or rotating gyroscope sensors, making highly accurate measurements difficult. In this presentation, a new type of input-output controller is introduced to reduce the sense-drive axes dependencies and compensate the imperfect manufacturing process, effectively reducing drift due to these. The main advantages of the controller are the robustness according to the Variable Structure System type of control law and the new gain adaptation technique. The controller is asymptotically stable and generates the control command for the sense axis force re-balancing fingers to synchronize the system and reduce cross-axis parameter dependencies.

   This work is joint with Jeremy Dawson, West Virginia High Technology Consortium Foundation, and Parviz Famouri, West Virginia University.

6. **Catherine Lebiedzik**, *Department of Mathematics, Wayne State University*

Hadamard Wellposedness for a Class of Non-Linear Shallow Shell Problems

This paper is concerned with the nonlinear shallow shell model introduced in 1966 by W .T. Koiter. We consider a new version of this model which is based upon the intrinsic shell modeling techniques introduced by Michel Delfour and Jean-Paul Zolésio. We show existence and uniqueness of both regular and weak solutions to the dynamical model and that the solutions are continuous with respect to the initial data. While existence and uniqueness of *regular solutions* to nonlinear dynamic shell equations has been known, full Hadamard wellposedness of *weak* solutions, as shown in this paper, is a new result which solves an old open problem in the field.

This work is joint with John Cagnol, Pôle Universitaire Léonard de Vinci, Paris, France, Irena Lasiecka, University of Virginia, and Richard Marchand, Slippery Rock University, Slippery Rock, PA.

7. **Olena Mul**, *Department of Mathematics, University of Aveiro, Portugal*

Some Applications of Partial Differential Equations for Analysis and Control of Hybrid Oscillation Systems

Many oscillation problems with a great relevance in mathematics and applications may be reduced to the study of systems of partial differential equations with variable coefficients. They can describe oscillations in very different systems with distributed and discrete parameters such as: transmission pipelines for lifting minerals from great depth; aerial ropeway systems; spacecrafts of big size; controlled machine units in modern apparatuses for materials processing or transportation; etc. Intensive oscillations in such hybrid systems are possible, but not desirable. Therefore, it is very important to obtain conditions when oscillations take place as well as to find some possibilities of oscillation control. Mathematically, the solutions of the PDEs often have to satisfy not only boundary conditions, but also some conjugation constraints, which appreciably complicate the problem. In the research, the numerical method of the normal fundamental systems and the asymptotical method are applied, which turn out to be effective for the considered problems. They allow us to determine complex eigenvalues of corresponding boundary problems and to obtain the dependencies of the oscillation frequencies on different physical parameters of the systems. The obtained results permit us to analyze the influence of different system parameters on oscillations. In this way, some possibilities of oscillation control for each mentioned system are found.

8. **Priya Ranjan**, *Elecrical Engineering/Distributed Systems, University of Maryland, College Park, and Intelligent Automation Inc.*

Moving boundary tracking using evolvable curves

In this work we extend the original scheme of tracking static boundaries by (Marthaler and Bertozzi, 2002) to track moving boundaries. Our schemes include explicit merging and splitting of boundaries under different application-specific factors. We illustrate that the original scheme coupled with an adaptive communication sensing paradigm based on distance from neighbors is practical and achieves the goal of tracking moving boundaries satisfactorily. Basic theoretical modeling for dynamic boundary tracking framework and stability results in terms of number of robots and communication bandwidth and extensive numerical simulation will also be presented. Finally, we will show some movies depicting the moving boundary tracking in action with limited number of cooperating robots. This system has many practical applications like oil spill tracking, poison plum detection, and containment swarming, etc.

This work is joint with Zachary Kulis and Vikram Manikonda, Intelligent Automation Inc., Rockville, Maryland.

9. **Moulay Rchid Sidi Ammi**, *Department of Mathematics, University of Aveiro, Portugal*

A Dead Oil Isotherm Optimal Control Problem

We are interested to study a system (P) of nonlinear partial differential equations resulting from traditional modeling of oil engineering within the framework of the mechanics of the continuous medium. The purpose of this paper is to establish existence and regularity of an optimal solution for this system. Moreover we derive necessary optimality conditions. The studied Problem (P) serves as a model of an incompressible biphasic flow in a porous medium, with applications in the industry of exploitation of hydrocarbons. To understand the optimal control problem we consider here, some words about the recovery of hydrocarbons are needed. At the time of the first run of a layer, the flow of the crude oil toward the surface is due to the energy stored in the gases under pressure or in the natural hydraulic system. To mitigate the consecutive decline of production and the decomposition of the site, water injections are carried out, well before the normal exhaustion of the layer. The water is injected through wells with high pressure, by pumps specially drilled with this end. The pumps allow the displacement of the crude oil toward the wells of production. The wells must be judiciously distributed, which gives rise to a difficult problem of optimal control: how to choose the best installation sites of the production wells? This is precisely the question we address in this work. Our main goal is to present a method to carry out the optimal control of (P). More precisely, the problem consists in seeking the admissible control parameters which minimize a given objective functional. The cost functional comprises all the important parameters that intervene in the processes. To study regularity of solutions which provide Gâteaux differentiability of the nonlinear operator corresponding to (P), we are forced to assume more regularity on the control as well as to impose compatibility conditions between initial and boundary conditions. Adequate choice of the functional cost to be minimized together with the hypotheses on the control, which provide regularity of the optimal solution, allow us to obtain the intended necessary optimality conditions. Here we make use of the Lagrangian method to obtain the optimality conditions. From the functional cost we get the Lagrangian, then we compute the Gâteaux derivative with respect to the various variables. This technique has been used with success by O. Bodart and R. Touzani for an optimal control problem of the induction heating, and by H.-C. Lee and T. Shilkin for the thermistor problem.

This work is joint with Delfim F. M. Toress, University of Aveiro, Portugal.

10. **Ilya Shvartsman**, *Department of Mathematics and Statistics, Miami University*

Minimax Approach to Feedback Control of Distributed Parameter Systems

This work concerns a minimax control design problem for a class of parabolic systems with nonregular boundary conditions and uncertain distributed perturbations under control and state constraints. The original motivation for this problem comes from an environmental problem of groundwater control, which has important applications to practical areas. A natural approach to control design of such uncertain systems is minimax synthesis, which guarantees the best system performance under the worst perturbations and ensures an acceptable behavior for any admissible perturbations.

This work is joint with B.S. Mordukhovich, Wayne State University.

11. **Daniel Toundykov**, *Department of Mathematics, University of Virginia*

Finite-dimensionality of the global attractor for a semilinear wave equation with localized dissipation, and a source of critical exponent

This work addresses long-term behavior of solutions to a semilinear damped wave equation with a source term. The dissipation is nonlinear and affects only a small sub-domain adjacent to a portion of the boundary; the source is modeled by the Nemytski operator associated to a nonlinear map whose order may include (in dimensions above 2) the corresponding critical Sobolev exponent (for the embedding $H^1 \to L^2$). We investigate existence of global attractors, their dimensionality and smoothness.

It is known that existence of attractors is strictly linked with asymptotic compactness of evolution trajectories, while at the level of critical exponents the compactness of Sobolev embeddings is lost and the extensive machinery developed for sub-critical settings no longer applies. The study of asymptotic properties under critical damping and sources is a challenging problem, and a comprehensive treatment of such models appeared only recently in the literature — in the work of I. Lasiecka and I. Chueshov.

The results so far, however, have dealt primarily with full interior or full boundary damping. In this case we investigate the issue of critical exponents combined with restricted dissipation, when the damping excludes a neighborhood of a boundary segment. We show that such a system is asymptotically smooth, and the fractal dimension of the global attractor is finite.

This work is joint with Irena Lasiecka, University of Virginia and Igor Chueshov, Kharkov National University, Ukraine.

12. **Amjad Tuffaha**, *Department of Mathematics, University of Virginia*

Control Problems in Fluid Structure Interaction

We consider a nonlinear model of fluid-structure interaction in 3 dimensions comprised of a Navier Stokes equation coupled through forces on an interface with an elastic equation. Two aspects of the model are studied (i) Well-posedness: Weak and Strong solutions, (ii) Feedback Boundary control strategies for the model.

This work is joint with Viorel Barbu, University of Iasi, Romania, Zoran Grujic, University of Virginia, and Irena Lasiecka, University of Virginia.

13. **Quan-Fang Wang**, *Department of Automation and Computer-Aided Engineering, The Chinese University of Hong Kong*

Optimal Control for Nonlinear Cahn-Hilliard Equations with Numerical Realization

As is well known, nonlinear Cahn-Hilliard (CH) equations describe a continuous model for phase transition in binary systems such as alloy, glasses and polymer-mixtures. This poster is to address theoretical and numerical issues for optimal control of distributed parameters system given by CH equations. For the performance index defined by quadratic criteria, a semi-discrete algorithm is constructed for minimization using finite element approach. The convergency and error estimation will be discussed. Numerical simulation interrupted the efficiency of proposed paradigm. I hope the poster will give a completely study for the CH systems.

14. **Weidong Zhu**, *Department of Mechanical Engineering, University of Maryland, Baltimore County*

Dynamics and Control of Distributed Structural Systems

In this poster we will describe some of our recent research results on the dynamics and control of distributed structural systems including: (1) a new dynamic stability theory for translating media with variable length and/or speed, and (2) control of high frequency modes of distributed structures. In the first area two types of stability problems are considered: stability of variable-length systems during extension and retraction, and stability of systems with periodically varying length and/or speed. New stability characteristics are identified for both systems from the energy viewpoint. In addition to stability analyses, free and forced response solutions using modal, wave, and finite difference methods are derived, and methods to dissipate the vibratory energies of the systems are addressed. A novel experimental method is developed to validate the uncontrolled and controlled lateral responses of a moving elevator cable with variable length in a high-rise elevator. This includes the design, analysis, and fabrication of a scaled elevator, experimental setup, and development of measurement and parameter estimation techniques. Experimental results show excellent agreement with the theoretical predictions. The application of the new methodology to the design of elevator systems is discussed. In the second area we will show the optimal location and damping coefficient of a discrete damper to dissipate all the modes of vibration of a translating or stationary, tensioned beam. The methodology can be applied to the design of power transmission lines and discrete controllers for distributed systems.

This work is joint with N.A. Zheng and Y. Chen, Department of Mechanical Engineering, University of Maryland, Baltimore County.