Name:______________________

MATH152

Quiz 5

date 06/30/2009

Total 60

Show all work legibly.

1. (20) Determine whether the integral converges or diverges. If it converges, evaluate the integral:

$$\int_{-\infty}^{\infty} e^{-2x}\,dx$$

The integral exists only if $\int_{-\infty}^{0} e^{-2x}\,dx$ and $\int_{0}^{\infty} e^{-2x}\,dx$ both exist.

$$\begin{aligned}\int_{-\infty}^{0} e^{-2x}\,dx &= \lim_{R\to-\infty} -\frac{e^{-2x}}{2}\Big|_{R}^{0} \\ &= \lim_{R\to-\infty} -\frac{1}{2}+\frac{e^{-R}}{2} \\ &= \infty\end{aligned}$$

$\int_{-\infty}^{\infty} e^{-2x}\,dx$ is divergent

2. (20) Determine whether the series converges or diveges. If it converges, compute the sum:

$$\sum_{k=2}^{\infty} \frac{4^k}{7^{k-1}}$$

$$\begin{aligned} \sum_{k=2}^{\infty} \frac{4^k}{7^{k-1}} &= \sum_{k=2}^{\infty} \frac{16}{7} (\frac{4}{7})^{k-2} \\ &= \frac{16/7}{1 - 4/7} \\ &= 16/3 \end{aligned}$$

since it is a geometric series with $a = 16/7$ and $r = 4/7$.

$\sum_{k=2}^{\infty} \frac{4^k}{7^{k-1}} = 16/3$

3. (20) Determine whether the series converges or diveges. If it converges, compute the sum:

$$\sum_{k=1}^{\infty} \frac{8}{k(k+2)}$$

$$\begin{aligned} \frac{8}{k(k+2)} &= \frac{A}{k} + \frac{B}{k+2} \\ 8 &= A(k+2) + Bk \end{aligned}$$

Substituting $k = -2$ and $k = 0$ give $A = 4$ and $B = -4$. Then

$$\begin{aligned} \frac{8}{k(k+2)} &= \sum_{k=1}^{\infty} \frac{4}{k} - \frac{4}{k+2} \\ &= 4/1 - 4/3 + 4/2 - 4/4 + 4/3 - 4/5 + 4/4 - 4/6 + 4/5 - 4/7 + \ldots \end{aligned}$$

Since the series is telescoping and $\frac{4}{k} - \frac{4}{k+2}$ approaches 0 as k approaches ∞, the sum of the series is equal to the sum of the only terms which do not eventually cancel, 4 and 2.

$$\sum_{k=1}^{\infty} \frac{8}{x(x+2)} = 6$$