Name:______________________

MATH152

Quiz 4 Solutions

date 06/23/2009

Total 60

Show all work legibly.

1. (20) Integrate: $\int e^{3x} \sin x \, dx$

   Integrate by parts, $u = e^{3x}, du = 3e^{3x}dx, dv = \sin x dx, v = -\cos x$

$$\int e^{3x} \sin x \, dx = -e^{3x} \cos x + 3 \int e^{3x} \cos x \, dx$$

   Now let $u = e^{3x}, du = 3e^{3x}dx, dv = \cos x dx, v = \sin x$

$$\begin{aligned} \int e^{3x} \sin x \, dx &= -e^{3x} \cos x + 3[e^{3x} \sin x - 3 \int e^{3x} \sin x \, dx] \\ 10 \int e^{3x} \sin x \, dx &= -e^{3x} \cos x + 3e^{3x} \sin x \\ \int e^{3x} \sin x \, dx &= \frac{-e^{3x} \cos x + 3e^{3x} \sin x}{10} + C \end{aligned}$$

2. (20) Integrate: $\int \tan^4 x \sec^6 x \, dx$

$$\begin{aligned}
\int \tan^4 x \sec^6 x \, dx &= \int \tan^4 x \sec^4 x \sec^2 x \, dx \\
&= \int \tan^4 x (\sec^2 x)^2 \sec^2 x \, dx \\
&= \int \tan^4 x (1 + \tan^2 x)^2 \sec^2 x \, dx \\
&= \int \tan^4 x (1 + 2\tan^2 x + \tan^4 x) \sec^2 x \, dx \\
&= \int [\tan^4 x + 2\tan^6 x + \tan^8 x] \sec^2 x \, dx \\
&= \frac{\tan^5 x}{5} + \frac{2\tan^7 x}{7} + \frac{\tan^9 x}{9} + C
\end{aligned}$$

3. (20) Find the partial fraction decomposition and integrate: $\int \frac{2x^2-5x+2}{x^3+x}\,dx$

$$\begin{aligned}
\frac{2x^2-5x+2}{x^3+x} &= \frac{2x^2-5x+2}{x(x^2+1} \\
&= \frac{A}{x} + \frac{Bx+C}{x^2+1} \\
2x^2-5x+2 &= A(x^2+1) + Bx^2 + Cx
\end{aligned}$$

Let $x = 0$ to get $A = 2$. It then follows by matching coefficients that $B = 0$ and $C = -5$. So

$$\begin{aligned}
\int \frac{2x^2-5x+2}{x^3+x}\,dx &= \int [\frac{A}{5} - \frac{5}{x^2+1}]\,dx \\
&= 2\ln|x| - 5\arctan x + C
\end{aligned}$$