

Name: \_\_\_\_\_

**MATH152**

Quiz 3 Solutions

date 06/16/2009

Total 60

Show all work legibly.

1. (20) Consider the region bounded by  $y = \sqrt{x}$ ,  $y = 2$ , and  $x = 0$ . Using the method of washers, compute the volume  $V$  of the solid formed by revolving the region about the line  $x = 4$ .

$$\begin{aligned} V &= \int_0^2 \pi(4)^2 dy - \int_0^2 \pi(4 - y^2)^2 dy \\ &= \int_0^2 16\pi dy - \int_0^2 \pi(16 - 8y^2 + y^4) dy \\ &= 16\pi y \Big|_0^2 - \pi \left( 16y - \frac{8}{3}y^3 + \frac{1}{5}y^5 \right) \Big|_0^2 \\ &= \pi(64/3 - 32 - 5) \\ &= \pi 225/15 \end{aligned}$$

2. (20) Consider the region bounded by  $y = 9 - x^2$  and  $y = 0$ . Using the method of cylindrical shells, compute the volume  $V$  of the solid formed by revolving the region about the line  $x = 5$ .

$$\begin{aligned} V &= 2\pi \int_{-3}^3 (5-x)(9-x^2) dx \\ &= 2\pi \left( \frac{x^4}{4} - \frac{5x^3}{3} - \frac{9x^2}{2} + 45x \right) \Big|_{-3}^3 \\ &= 4\pi [45(3) - 5(3^3)/3] \\ &= 360\pi \end{aligned}$$

3. (20) Compute the exact arc length  $s$  of  $f(x) = 6x^{3/2} + 3$  on  $0 \leq x \leq 2$ . You can leave exponents in your answer.

$$\begin{aligned} f'(x) &= 9x^{1/2} \\ s &= \int_0^2 \sqrt{1 + 81x} \, dx \\ &= \frac{1}{81} \int_1^{163} \sqrt{u} \, du \\ &= \frac{2}{243} u^{3/2} \Big|_1^{163} \\ &= \frac{2}{243} [163^{3/2} - 1] \end{aligned}$$

where the third line follows from the substitution  $u = 1 + 81x$ .