

Name: _____

MATH152

Quiz 2 Solutions

date 06/09/2009

Total 80

Show all work legibly.

1. (20) Find the derivative of $f(x) = \int_3^{2x^2+5} \ln t \, dt$ for $2x^2 + 5 \geq 3$.

The answer follows from Part II of the Fundamental Theorem of Calculus and the chain rule.

$$f'(x) = 4x \ln(2x^2 + 5)$$

2. (20) Find the general antiderivative: $\int \frac{2\sqrt{\ln x}}{x} dx$

$$u = \ln x$$

$$du = (1/x)dx$$

$$\begin{aligned} \int \frac{2\sqrt{\ln x}}{x} dx &= 2 \int u^{1/2} du \\ &= \frac{4}{3} u^{3/2} + C \end{aligned}$$

$$\int \frac{2\sqrt{\ln x}}{x} dx = \frac{4}{3} (\ln x)^{3/2} + C$$

3. (20) Evaluate the integral exactly: $\int_{\ln 3}^{\ln 9} \frac{e^x}{1 + e^{2x}} dx$

$$u = e^x$$

$$du = e^x dx$$

$$\begin{aligned} \int_{\ln 3}^{\ln 9} \frac{e^x}{1 + e^{2x}} dx &= \int_3^9 \frac{1}{1 + u^2} du \\ &= \arctan(u) \Big|_3^9 \\ &= \arctan(9) - \arctan(3) \end{aligned}$$

$$\int_{\ln 3}^{\ln 9} \frac{e^x}{1 + e^{2x}} dx = \arctan(9) - \arctan(3)$$

4. (20) Use properties of the natural logarithm to reduce then differentiate (i.e. don't use the quotient

rule): $\frac{d}{dx} \left(\ln \sqrt{\frac{x}{x^3 + 9}} \right)$

$$\begin{aligned} \ln \sqrt{\frac{x}{x^3 + 9}} &= \frac{1}{2} \ln \frac{x}{x^3 + 9} \\ &= \frac{1}{2} \ln x - \frac{1}{2} \ln x^3 + 9 \end{aligned}$$

So

$$\frac{d}{dx} \left(\ln \sqrt{\frac{x}{x^3 + 9}} \right) = \frac{1}{2x} - \frac{3x^2}{2(x^3 + 9)}$$

$$\frac{d}{dx} \left(\ln \sqrt{\frac{x}{x^3 + 9}} \right) = \frac{1}{2x} - \frac{3x^2}{2(x^3 + 9)}$$