

Name: _____

MATH152

Quiz 1

date 06/05/2008

Solutions

Total 60

Show all work legibly.

1. (20) Find the general antiderivative: $\int \frac{\cos x}{\sin x} dx$

Since the derivative of $f(x) = \sin x$ is $\cos x$, the integrand has the form $\frac{f'(x)}{f(x)}$. So by Corollary 1.2 of Section 4.1, the integral is equal to $\ln|\sin x| + C$.

2. (20) Compute the sum: $\sum_{i=1}^{50} (3i^2 + 5i + 1)$

$$\begin{aligned}\sum_{i=1}^{50} (3i^2 + 5i + 1) &= 3 \sum_{i=1}^{50} i^2 + 5 \sum_{i=1}^{50} i + \sum_{i=1}^{50} 1 \\ &= 3 \frac{50(51)(101)}{6} + 5 \frac{50(51)}{2} + 50 \\ &= (25)(51)(101) + (125)(51) + 50\end{aligned}$$

No need to compute further.

3. (20) Use Riemann sums and a limit to compute the exact area A under the curve $f(x) = x^2 + 4x$ on $[1, 3]$

$$\Delta x = \frac{3-1}{n} = \frac{2}{n}, x_i = 1 + \frac{2i}{n}$$

$$\begin{aligned} A &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\left(1 + \frac{2i}{n}\right)^2 + \left(4 + \frac{8i}{n}\right) \right] \frac{2}{n} \\ &= \lim_{n \rightarrow \infty} \frac{2}{n} \sum_{i=1}^n \left[5 + \frac{12i}{n} + \frac{4i^2}{n^2} \right] \\ &= \lim_{n \rightarrow \infty} \left[\frac{2}{n} \sum_{i=1}^n 5 + \frac{24}{n^2} \sum_{i=1}^n i + \frac{8}{n^3} \sum_{i=1}^n i^2 \right] \\ &= \lim_{n \rightarrow \infty} \left[\frac{10n}{n} + \frac{24}{n^2} \frac{n(n+1)}{2} + \frac{8}{n^3} \frac{n(n+1)(2n+1)}{6} \right] \\ &= 10 + 12 + 8/3 \\ &= 74/3 \end{aligned}$$