1. (25 pts) Given the matrix \( A = \begin{pmatrix} 2 & 2 \\ -2 & 2 \end{pmatrix} \)

   a) (5 pts) Compute the characteristic polynomial of \( A \).

   b) (5 pts) Find the eigenvalues of \( A \).

   c) (10 pts) Find the eigenvectors of \( A \).

   d) (5 pts) Find a matrix \( P \) such that \( P^{-1}AP \) is diagonal and compute \( P^{-1}AP \).

2. (25 pts) Given the matrix \( A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 2 & 1 \\ -1 & -2 & 5 & 1 \end{pmatrix} \)

   a) (6 pts) Compute \( \text{rank}(A) \).

   b) (6 pts) Find a basis of the column space of \( A \).

   c) (13 pts) Find a basis of the null space of \( A \).

3. (25 pts) Prove that if \( A \) has an eigenvalue \( \lambda \) then the matrix \( A^3+2A^2+A \) has an eigenvalue \( \lambda^3+2\lambda^2+\lambda \).

4. (25 pts) Multiple Choice

   a) The rank of a 3x3 non-zero matrix is greater than zero.
      ALWAYS/NEVER/SOMETIMES

   b) If \( B=\{b_1,b_2,b_3\} \) is a basis of \( \mathbb{R}^3 \) then \( \{Ab_1,Ab_2,Ab_3\} \) is a basis of \( \mathbb{R}^3 \).
      ALWAYS/NEVER/SOMETIMES

   c) A change of coordinates matrix is invertible: ALWAYS/NEVER/SOMETIMES

   d) Similar matrices have the same eigenvectors. ALWAYS/NEVER/SOMETIMES

   e) The matrix \( A \) is not diagonalizable. The product of the eigenvalues of \( A \) is equal to the determinant of \( A \). ALWAYS/NEVER/SOMETIMES