1) (20 pts) Use Gaussian Elimination to compute $A^{-1}$ where
\[
A = \begin{pmatrix}
3 & 1 & 2 & 1 \\
1 & 2 & 1 & -1 \\
2 & 1 & 3 & -1 \\
1 & 1 & -2 & 2
\end{pmatrix}
\]

2) (40 pts) If $A = \begin{pmatrix}1 & 2 \\ 2 & 1 \end{pmatrix}$ find $B$ such that $AB = BA$:
   a) Find a single matrix $B$ such that $AB = BA$.
   b) Find all matrices $B$ such that $AB = BA$.
   c) Prove that if $B$ and $C$ are such that $AB = BA$ and $AC = CA$ then if the matrix $D$ is a linear combination of $B$ and $C$ then $AD = DA$.

3) (20 pts) Given the matrix $A = \begin{pmatrix}1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$
   a) Produce an LU factorization of $A$ (without pivoting)
   b) Use the LU factorization to solve $Ax = b$ for $x$ where $b = \begin{pmatrix}1 \\ 3 \\ 2\end{pmatrix}$

4) (20 pts) Prove that the inverse of a 2x2 elementary matrix is an elementary matrix of the same type. (Prove this separately for each type of elementary matrix.)