1. (25 pts) Given \( A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & 4 & -2 \end{pmatrix} \), \( b = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix} \) and \( c = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} \).

   a) (7 pts) Bring matrix \( A \) to echelon form.
   b) (2 pts) Which columns of \( A \) are pivot columns?
   c) (16 pts) Find all solutions to:
      i. (8 pts) \( Ax = b \)
      ii. (8 pts) \( Ax = c \)

2. (25 pts) Consider the set of vectors \( \vec{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \), \( \vec{v}_2 = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix} \), \( \vec{v}_3 = \begin{pmatrix} 2 \\ -6 \\ 1 \end{pmatrix} \).

   a) (10 pts) Is the set \( \{ \vec{v}_1, \vec{v}_2, \vec{v}_3 \} \) linearly independent? If not, find a non-trivial linear relation between the members of the set.
   b) (15 pts) Can \( \vec{b} = \begin{pmatrix} 2 \\ -8 \\ -1 \end{pmatrix} \) be expressed as a linear combination of \( \{ \vec{v}_1, \vec{v}_2, \vec{v}_3 \} \)? If so, write \( b \) as a linear combination of \( \{ \vec{v}_1, \vec{v}_2, \vec{v}_3 \} \) in all possible ways.

3. (25 pts) Given the system of equations
   \[
   \begin{align*}
   x + 2y + 2z + 6w &= -1 \\
   -2x - 4y - 2z - 9w &= 3 \\
   x + 2y + 4z + 12w &= -3
   \end{align*}
   \]

   a) (5 pts) Express this system as an augmented matrix.
   b) (10 pts) Bring this matrix to reduced echelon form.
   c) (10 pts) Is the system consistent? If so, write down all solutions of the original system.

4. (25 pts) ALWAYS True, NEVER True or SOMETIMES True (5 pts per question)

   a) The matrix corresponding to the linear transform
      \[
      \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x + y \\ x - y \end{pmatrix}
      = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix}
      \]
   b) A linear transformation from \( \mathbb{R}^5 \) to \( \mathbb{R}^5 \) is 1-to-1.
   c) The span of three vectors \( \{ \vec{v}_1, \vec{v}_2, \vec{v}_3 \} \) is equal to the span of the three vectors \( \{ \vec{v}_1 + \vec{v}_2, \vec{v}_2 + \vec{v}_3, \vec{v}_3 + \vec{v}_1 \} \).
   d) A set of 10 vectors in \( \mathbb{R}^5 \) is linearly dependent.
   e) A 1-to-1 linear transformation \( T: \mathbb{R}^3 \rightarrow \mathbb{R}^5 \) is onto.