1. (25 pts) Given the system of equations
\[
\begin{align*}
x + y + z &= 1 \\
2x + y - 2z &= 3 \\
2x + 2y + z &= 3
\end{align*}
\]
a) (5 pts) Express this system as an augmented matrix
b) (10 pts) Bring this matrix to reduced row echelon form.
c) (10 pts) Write down all solutions of the original system.

2. (25 pts) Given
\[
A = \begin{pmatrix}
1 & 3 & 5 \\
2 & 2 & 2 \\
3 & 1 & -1
\end{pmatrix}
\]
a) (9 pts) Bring this matrix to row echelon form.
b) (4 pts) Which columns are pivot columns?
c) (3 pts) Does \( Ax = 0 \) have non-trivial solutions?
d) (9 pts) Write down all solutions of \( Ax = 0 \).

3. (25 pts) Is \( \vec{u} = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} \) in the span of \( \vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \vec{v}_3 = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} \)?
If so, write \( u \) as a linear combination of \( v_1, v_2 \) and \( v_3 \).

4. (25 pts) True or False (5 pts per correct answer, -2 pts per incorrect answer)
a) A homogeneous linear system is always consistent.
b) The matrix \( \begin{pmatrix} 1 & 2 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \) has 3 pivot columns.
c) A system of \( n \) linear equations in \( n \) variables has at most \( n \) solutions.
d) If \( Ax = 0 \) has the trivial solution then \( A \) has no pivot columns.
e) A linear map from \( \mathbb{R}^3 \) to \( \mathbb{R}^3 \) is always onto.